GCE Examinations

Advanced Subsidiary / Advanced Level

Further Pure Mathematics
Module FP3

Paper A

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks should be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.



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FP3 Paper A – Marking Guide

1. *(a)* $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 0 & 1 \\ 2 & -1 & -2 \end{vmatrix}$

$= \mathbf{i}(0+1) - \mathbf{j}(-2-2) + \mathbf{k}(-1-0) = \mathbf{i} + 4\mathbf{j} - \mathbf{k}$ M1 A2

(b) $d = \left|\dfrac{(-4\mathbf{i}+\mathbf{j}-5\mathbf{k}).(\mathbf{i}+4\mathbf{j}-\mathbf{k})}{\sqrt{1+16+1}}\right|$ M1 A1

$= \left|\dfrac{-4+4+5}{\sqrt{18}}\right| = \dfrac{5}{\sqrt{18}}$ or $\frac{5}{6}\sqrt{2}$ A1 **(6)**

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2. assume true for $n = k$ $\therefore \sum_{r=1}^{k} (r^2+1)r! = k(k+1)!$

$\therefore \sum_{r=1}^{k+1} (r^2+1)r! = k(k+1)! + [(k+1)^2+1](k+1)!$ M1 A1

$= (k+1)!(k+k^2+2k+2) = (k+1)!(k^2+3k+2)$ M1

$= (k+1)!(k+2)(k+1) = (k+1)[(k+1)+1]!$ A1

$\therefore$ true for $n = k+1$ if true for $n = k$

if $n = 1$ $\sum_{r=1}^{n} (r^2+1)r! = 2 \times 1! = 2$; $n(n+1)! = 1 \times 2! = 2$ $\therefore$ true for $n = 1$ B1

$\therefore$ by induction true for $n \in \mathbb{Z}^+$ A1 **(6)**

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3. *(a)* $z^3 = -27$ $\therefore (re^{i\theta})^3 = 27e^{i\pi}$ M1

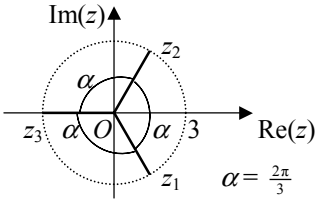
$r^3 = 27$ so $r = 3$ A1

$3\theta = 2n\pi + \pi$ M1

$n = -1, 0, 1$ gives $\theta = -\frac{\pi}{3}, \frac{\pi}{3}, \pi$ A1

$\therefore z_1 = 3e^{-i\frac{\pi}{3}}, z_2 = 3e^{i\frac{\pi}{3}}, z_3 = 3e^{i\pi}$ A1

(b)



$\alpha = \frac{2\pi}{3}$ B2 **(7)**

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4. *(a)* $\begin{vmatrix} 2-\lambda & a \\ 2 & b-\lambda \end{vmatrix} = 0$ $\therefore (2-\lambda)(b-\lambda) - 2a = 0$ M1 A1

$\lambda_1 = -2$ gives $4b - 2a + 8 = 0$

$\lambda_2 = 3$ gives $-b - 2a + 3 = 0$ M1

solve simul. giving $a = 2, b = -1$ A1

(b) $\lambda_1 = -2$, $\begin{pmatrix} 4 & 2 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$, $4x + 2y = 0$, $\therefore y = -2x$ $\therefore$ eigenvector $k\begin{pmatrix} 1 \\ -2 \end{pmatrix}$ M1 A1

$\lambda_1 = 3$, $\begin{pmatrix} -1 & 2 \\ 2 & -4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$, $-x + 2y = 0$, $\therefore x = 2y$ $\therefore$ eigenvector $k\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ A1

(c) $\mathbf{P} = \dfrac{1}{\sqrt{5}}\begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix}$ M1 A1 **(9)**

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5. $x_0 = -1, y_0 = 1, \left(\frac{dy}{dx}\right)_0 = 1;\ 2\left(\frac{d^2y}{dx^2}\right)_0 - 4 + 2 = 0 \ \therefore \left(\frac{d^2y}{dx^2}\right)_0 = 1$ M1 A1

$(1+x^2)\frac{d^3y}{dx^3} + 2x\frac{d^2y}{dx^2} + 4x\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 2\frac{dy}{dx} = 0$ M1 A1

$(1+x^2)\frac{d^3y}{dx^3} + 6x\frac{d^2y}{dx^2} + 6\frac{dy}{dx} = 0$ A1

$2\left(\frac{d^3y}{dx^3}\right)_0 - 6 + 6 = 0 \ \therefore \left(\frac{d^3y}{dx^3}\right)_0 = 0$ A1

$(1+x^2)\frac{d^4y}{dx^4} + 2x\frac{d^3y}{dx^3} + 6x\frac{d^3y}{dx^3} + 6\frac{d^2y}{dx^2} + 6\frac{d^2y}{dx^2} = 0$ M1 A1

$2\left(\frac{d^4y}{dx^4}\right)_0 + 0 + 12 = 0 \ \therefore \left(\frac{d^4y}{dx^4}\right)_0 = -6$ A1

$y = 1 + 1(x+1) + \frac{1}{2!}(x+1)^2 - \frac{6}{4!}(x+1)^4 + \ldots$ M1

$y = 1 + (x+1) + \frac{1}{2}(x+1)^2 - \frac{1}{4}(x+1)^4 + \ldots$ A1 **(11)**

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6. $\frac{y_1 - 2y_0 + y_{-1}}{0.01} = x_0 \times \frac{y_1 - y_{-1}}{0.2} + y_0^2$ M1 A1

$20y_1 - 40y_0 + 20y_{-1} = x_0y_1 - x_0y_{-1} + 0.2y_0^2$ M1

$y_1(20 - x_0) = 40y_0 - 20y_{-1} - x_0y_{-1} + 0.2y_0^2$ M1

$y_1 = \frac{40y_0 - 20y_{-1} - x_0y_{-1} + 0.2y_0^2}{20 - x_0}$ A1

$x_{-1} = 0.1, x_0 = 0.2, x_1 = 0.3;\ y_{-1} = 1.2, y_0 = 0.9, y_1 = ?$

$y_1 = \frac{36 - 24 - 0.24 + 0.162}{19.8} = 0.60212\ldots = 0.602$ (3sf) M1 A2

$y_2 = \frac{40y_1 - 20y_0 - x_1y_0 + 0.2y_1^2}{20 - x_1}$ M1

$x_0 = 0.2, x_1 = 0.3, x_2 = 0.4;\ y_0 = 0.9, y_1 = 0.60212\ldots, y_2 = ?$

$y_2 = \frac{24.08\ldots - 18 - 0.27 + 0.0725\ldots}{19.7} = 0.29885\ldots = 0.299$ (3sf) M1 A1 **(11)**

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7. *(a)* $\det \mathbf{M} = 2(8-3k) - 1(2k+3) + 1(k^2+4) = k^2 - 8k + 17$ M1 A1

(b) $\det \mathbf{M} = (k-4)^2 - 16 + 17 = (k-4)^2 + 1$ M1

$(k-4)^2 \geq 0 \ \therefore \det \mathbf{M} > 0 \ \therefore$ **M** non-singular for all real k A1

(c) $k = 3, \ \mathbf{M} = \begin{pmatrix} 2 & 1 & 1 \\ 3 & 4 & 3 \\ -1 & 3 & 2 \end{pmatrix}, \ \det \mathbf{M} = 2$ B1

matrix of cofactors: $\begin{pmatrix} -1 & -9 & 13 \\ 1 & 5 & -7 \\ -1 & -3 & 5 \end{pmatrix}$ M1 A1

$\therefore \mathbf{M}^{-1} = \frac{1}{2}\begin{pmatrix} -1 & 1 & -1 \\ -9 & 5 & -3 \\ 13 & -7 & 5 \end{pmatrix}$ A1

(d) $\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \mathbf{M}^{-1}\begin{pmatrix} 0 \\ 3 \\ 5 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} -1 & 1 & -1 \\ -9 & 5 & -3 \\ 13 & -7 & 5 \end{pmatrix}\begin{pmatrix} 0 \\ 3 \\ 5 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} -2 \\ 0 \\ 4 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix}$

$a = -1, \ b = 0, \ c = 2$ M1 A2 **(11)**

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8. *(a)* $w(\mathrm{i}z - 1) = z + 1; \quad \mathrm{i}wz - w = z + 1$ M1

$z(\mathrm{i}w - 1) = w + 1 \ \therefore \ z = \frac{w+1}{\mathrm{i}w-1}$ A1

$|z| = 1 \ \therefore \ |w+1| = |\mathrm{i}w - 1|$ M1

$|w+1| = |\mathrm{i}(w+\mathrm{i})| = |\mathrm{i}||w+\mathrm{i}| = |w+\mathrm{i}|$

$\therefore$ perp. bisector of -1 and $-\mathrm{i} \ \therefore \ u = v$ M1 A1

(b) $\mathrm{Im}\, z = 0 \ \therefore \ y = 0$ so $u + \mathrm{i}v = \frac{x+1}{\mathrm{i}x-1}$ M1

$(u + \mathrm{i}v)(-1 + \mathrm{i}x) = x + 1$

$-u - vx + \mathrm{i}(ux - v) = x + 1$ M1

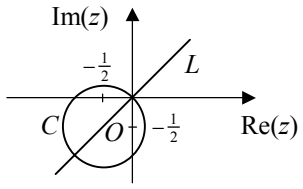
$-u - vx = x + 1 \ \therefore \ x = \frac{-u-1}{1+v}; \quad ux - v = 0 \ \therefore \ x = \frac{v}{u}$ A1

$\therefore \ \frac{-u-1}{1+v} = \frac{v}{u}; \ -u^2 - u = v + v^2$ M1

giving $(u + \frac{1}{2})^2 + (v + \frac{1}{2})^2 = \frac{1}{2}$ A1

$\therefore$ circle, centre $-\frac{1}{2} - \frac{1}{2}\mathrm{i}$, radius $\frac{1}{\sqrt{2}}$ A1

(c)



B3 **(14)**

Total **(75)**

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Performance Record – FP3 Paper A

Question no.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	Total
Topic(s)	vectors	proof by induction	complex nos.	matrices, eigenvals.	Taylor series soln. of diff. eqn.	step-by-step soln. of diff. eqn.	matrices, inverse	complex trans.	
Marks	6	6	7	9	11	11	11	14	75
Student									

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